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SELF-EXCITING POTENTIAL FLUID FLOW, SURROUNDED BY INHOMOGENEITIES, NEAR A CIRCULAR GRID OF PROFILES

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When a fluid flows through the grid of a turbine machine, in many cases, self-excitation of the velocity field surrounded by inhomogeneities, rotating in the direction of rotation of the grid, occurs. Such phenomena include a rotating discontinuity, which arises in certain regimes in axial turbine machines. In radial grids of centrifugal fans, the rotation of the velocity field was noted and described by Zhukovskii [1]. Recently, a similar phenomenon was also observed while studying fluid flow through a circular grid [2]. The surrounding nonuniformity of the velocity field in [2] was modeled by a displacement of a vortex source from the center of the grid. However, the mechanism for the motion of the vortex source was not examined.

In this paper, the indicated model of fluid flow through a planar circular grid is closed with the help of the equations of motion of the vortex source in the velocity field, perturbed by the profiles of the grid. In addition, the problem of self-excitation of the surrounding nonuniformity is reduced to the problem of the instability of the motion of the vortex source.

1. Experiment. The experiment was performed in a flow channel, consisting of an open tank with diameter 2 m and height 0.8 m (Fig. 1). The tank 1 contained a disk 2 with an aperture at the center, in which a diffuser 3 was inserted. A rod 4, on which a derive shaft 5 is mounted, rotating with the grid 6, was placed on the disk 2. The shaft was rotated by an electrical motor 7 via belt drive 8.

The flow was visualized by introducing confetti into the flow. Figure 2 presents photographs which were made by a camera in a fixed position (Fig. 2a) and rotating synchronously with the grid (Fig. 2b). The photographs clearly show the circular nonuniformity of the velocity field, which is manifested, for example, in the different angles of flow onto the profile. It is noted that this nonuniformity rotates with ≈ 50 times lower angular velocity than the rotational velocity of the grid.

2. Formulation of the Problem. We shall examine the two-dimensional flow of an ideal incompressible fluid through a circular grid, uniformly rotating with angular velocity ω (Fig. 3). As is well known, the fluid

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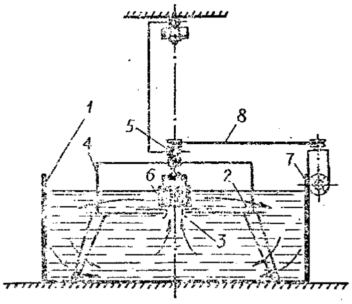

Fig. 1

flow onto the circular grid is usually modeled by a vortex source, situated at the center of the grid. We shall assume that a random perturbation displaces it from the center into some position ε_0. Then, the vortex source begins to move with the velocity of the fluid at the point $\varepsilon(t)$, with which its position coincides at the given moment, i.e.,

$$d\varepsilon/dt = v(\varepsilon). \tag{2.1}$$

The following question arises: does there exist a stable limit cycle of motion of the vortex source satisfying Eq. (2.1)? We shall solve this problem assuming that the absolute motion of the fluid outside the vortex source, outside the profiles, and outside the vortex wakes, converging with the profiles, is a potential flow. Then, the complex velocity of flow of the fluid can be determined from the expression

$$\bar{v}(z) = \frac{q - i\Gamma_0}{2\pi(z-\varepsilon)} + \frac{\partial\varphi}{\partial x} - i\frac{\partial\varphi}{\partial y},$$

where q and Γ_0 are parameters determining the intensity of the vortex source, and φ is a harmonic function satisfying the following boundary conditions:

condition that the fluid does not penetrate through the profile

$$\frac{\partial\varphi}{\partial\nu} = v_\nu^{(n)} - \frac{1}{2\pi}\,\mathrm{Re}\left[\frac{(q - i\Gamma_0)\,\nu_0^{(n)}}{z-\varepsilon}\right], \quad (x, y) \in L_n \quad (n = 1, 2, \ldots, N), \tag{2.2}$$

where $\nu_0^{(n)}$ is the unit outer normal to the contour of the n-th profile L_n; $v_\nu^{(n)}$ is the normal component of the velocity of points on the contour of the n-th profile; N is the number of profiles in the grid; the dynamic and kinematic conditions in the vortex wakes

$$[p] = 0, \ [\partial\varphi/\partial\nu_1] = 0, \ (x, y) \in \mathscr{L}_n \ (n = 1, 2, \ldots, N),$$

where p is the fluid pressure; ν_1 is the orientation of the normal to the lines of the contact discontinuity $\mathscr{L}_n$, simulating the vortex wakes; the condition at infinity

$$\lim_{|z|\to\infty} \nabla\varphi = 0;$$

the Kutta–Zhukovskii conditions

$$\Delta\varphi < \infty, \ (x, y) \in c_n \ (n = 1, 2, \ldots, N),$$

where c_n is the coordinate of the rear edge of the n-th profile.

3. Quasistationary Approximation for the Simplest Model of a Grid. To determine the velocity field of the fluid flow in the first approximation, we shall model the profiles of the grid by point vortices, situated on one fourth of the chord, and we shall satisfy the condition of impenetrability of the profile at points of the profiles at distances 3/4 of the chord from the tip.

In the quasistationary approximation, the complex velocity of the fluid flow for this model has the expression

$$\bar{v}(z) = \frac{q - i\Gamma_0}{2\pi}\frac{1}{z-\varepsilon} + \frac{1}{2\pi i}\sum_{n=1}^{N}\frac{\Gamma_n}{z - z_n}, \tag{3.1}$$

where $z_n = r\exp[i(\theta_n + \omega t)]$ are the coordinates of the positions of the vortices on the profile; θ_n is their angular coordinate; for $t = 0$ $\theta_n + (2\pi/N)(n-1)$ are determined from condition (2.2):

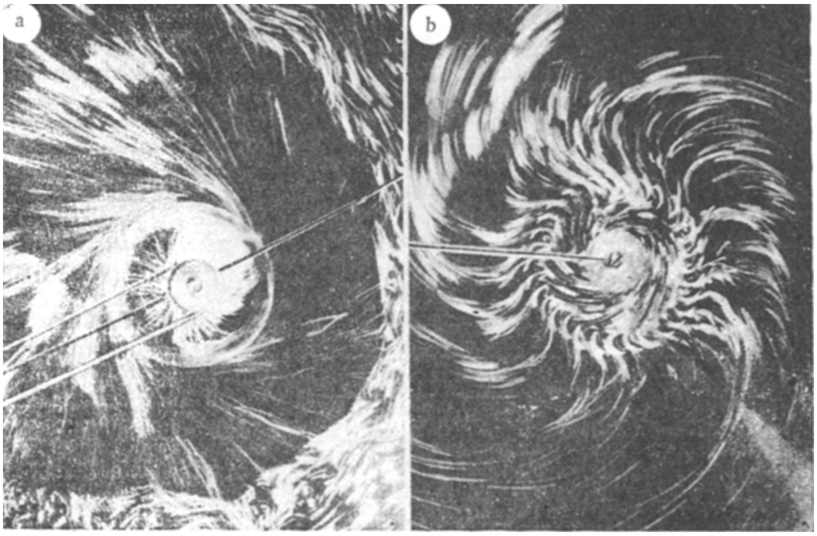

Fig. 2

$$\operatorname{Re}\,[\bar{v}(z_m)v_0(z_m)] = \omega R \sin \alpha_1 \;\; (m = 1, 2, \ldots, N), \tag{3.2}$$

where $z_m = R \exp[i(\varphi_m + \omega t)]$ are the coordinates of the control points of the profile; $\nu_0(z_m) = \exp[i(\alpha_m + \omega t)]$ are the unit normals to the profiles at these points; $\varphi_m = (2\pi/N)(m-1)$; $\alpha_m = \alpha_1 + \varphi_m$. We shall represent the coordinate of the position of the vortex source ε in the form

$$\varepsilon = \bar{r}_{\varepsilon} e^{i\sigma}, \;\; \sigma = \omega t - \sigma_1(t). \tag{3.3}$$

Then condition (3.2) including (3.1) transforms as follows:

$$\operatorname{Re}\left\{\frac{(q - i\Gamma_0)\, e^{i\alpha_1}}{1 - \bar{\varepsilon}\bar{r} e^{-i(\varphi_m + \sigma_1)}} - i e^{i\alpha_1} \sum_{n=1}^{N} \frac{\Gamma_n}{1 - \bar{r} e^{i(\theta_n - \varphi_m)}}\right\} = 2\pi\omega R^2 \sin \alpha_1 \tag{3.4}$$

$$(m = 1, 2, \ldots, N; \;\; \bar{r} = r/R).$$

The solution of the system (3.4) can be found in explicit form, if the values of the intensities of the circulations Γ_n sought are represented as trigonometric polynomials:

$$\Gamma_n = \gamma_0 + \sum_{k=1}^{N-1} (a_k \cos k\theta_n + b_k \sin k\theta_n), \tag{3.5}$$

where γ_0 is the stationary circulation, which is identical for all profiles; $a_k(t)$, $b_k(t)$ are functions of time determining the nonstationary component of the circulations arising around the profiles due to displacement of the vortex source.

In reality, expanding the fractions entering into expression (3.4), in the converging series and summing these series, we obtain from (3.4) using (3.5)

$$\operatorname{Re}\left\{\frac{(q - i\Gamma_0)\, e^{i\alpha_1}}{1 - (\bar{\varepsilon}\bar{r} e^{-i\sigma_1})^N}\left[1 + \sum_{k=1}^{N-1} (\bar{\varepsilon}\bar{r})^k e^{-ik(\sigma_1 + \varphi_m)}\right] - \frac{iN e^{i\alpha_1}}{2\left(1 - \bar{r}^N e^{i\theta_1 N}\right)}\left[2\gamma_0 + \sum_{k=1}^{N-1} \bar{r}^{N-k} e^{i(k\varphi_m + N\theta_1)} (a_k - ib_k) + \right.\right.$$
$$\left.\left. + \sum_{k=1}^{N-1} \bar{r}^k e^{-ik\varphi_m} (a_k + ib_k)\right]\right\} = 2\pi\omega R^2 \sin \alpha_1 \;\; (m = 1, 2, \ldots, N). \tag{3.6}$$

After appropriate transformations we have from (3.6) the system of equations

$$\sum_{k=0}^{N-1} (B_k \cos k\varphi_m + C_k \sin k\varphi_m) = 2\pi\omega R^2 \sin \alpha_1 \;\; (m = 1, 2, \ldots, N), \tag{3.7}$$

from which it follows that

$$B_0 = 2\pi\omega R^2 \sin \alpha_1, \; B_k = C_k = 0 \;\; (k \neq 0). \tag{3.8}$$

From relations (3.8), the values of γ_0 and the functions $a_k(t)$, $b_k(t)$ are determined explicitly.

To simplify the calculations, we shall examine the limiting case $N \to \infty$. Then, we obtain from (3.8)

$$\gamma_0 = (1/N)(2\pi\omega R^2 - \Gamma_0 - q \operatorname{ctg} \alpha_1),\ a_k = -(2\bar{\varepsilon}^k/N)(q \sin k\sigma_1 + \Gamma_0 \cos k\sigma_1),\ b_k = -(2\bar{\varepsilon}^k/N)(q \cos k\sigma_1 - \Gamma_0 \sin k\sigma_1). \tag{3.9}$$

Substituting expressions (3.9) into Eq. (3.5), we find

$$\Gamma_n = \gamma_0 - \frac{2}{N} \sum_{k=1}^{N-1} \bar{\varepsilon}^k [q \sin k(\sigma_1 + \theta_n) + \Gamma_0 \cos k(\sigma_1 + \theta_n)]. \tag{3.10}$$

Thus, the complex velocity of the fluid flow can be determined from Eq. (3.1) taking into account (3.10) as a function of the position of the vortex source. In particular, at the point itself it will equal

$$\bar{v}(\varepsilon) = \frac{1}{2\pi i} \sum_{n=1}^{N} \frac{\Gamma_n}{\varepsilon - \bar{z}_n} = \frac{e^{-i\omega t}}{2\pi i r} \sum_{n=1}^{N} \frac{1}{\varepsilon e^{-i\sigma_1} - e^{i\theta_n}} \left\{\gamma_0 - \frac{2}{N} \sum_{k=1}^{N-1} \bar{\varepsilon}^k [q \sin k(\sigma_1 + \theta_n) + \Gamma_0 \cos k(\sigma_1 + \theta_n)]\right\}. \tag{3.11}$$

Expanding the fraction entering into expression (3.11), in a converging series and summing it, we find

$$\bar{v}(\varepsilon) = -\frac{\exp[-i(\omega t - \sigma_1)]}{\pi r}(q + i\Gamma_0)\frac{\bar{\varepsilon}}{1 - \bar{\varepsilon}^2}.$$

Substituting the value of the velocity, complex conjugate to $\bar{v}(\varepsilon)$, in the equation of motion of the vortex source (2.1) and separating its real and imaginary parts, using (3.3), we obtain

$$d\bar{\varepsilon}/dt = -q\bar{\varepsilon}/(\pi r^2(1 - \bar{\varepsilon}^2)); \tag{3.12}$$

$$\omega_0 = d\sigma/dt = \Gamma_0/(\pi r^2(1 - \bar{\varepsilon}^2)), \tag{3.13}$$

where ω_0 is the rate of change of the angular coordinate of the vortex source. It follows from Eq. (3.12) that the solution of the problem stated in the quasistationary approximation for $\bar{\varepsilon} < 1$ does not have a limit cycle.

4. Taking into Account the Effect of Vortex Wakes. As will be evident from what follows, similarly to the system (3.12) and (3.13) in the limiting case examined $N \to \infty$, the corresponding system of ordinary differential equations is also autonomous when the influence of vortex wakes is included. It follows from here that the limit cycle, if it exists, will represent a circle with center at the origin of coordinates ($\bar{\varepsilon}$ = const). In this case, the vortex source will move along the trajectory with constant angular velocity ω_0, i.e., in expression (3.3) the quantity $\sigma = \omega_0 t$. It also follows from (3.3) that

$$\sigma_1(t) = \omega_1 t, \text{where } \omega_1 = \omega - \omega_0 = \text{const}. \tag{4.1}$$

In accordance with expression (3.6) and using (4.1), the nonstationary component of the complex velocity of fluid flow, induced by the vortex source at the control points of the profiles, can be represented as a sum of harmonics

$$\bar{v}(z_m) = \frac{q - i\Gamma_0}{2\pi r} \sum_{k=1}^{N-1} (\bar{\varepsilon}\bar{r} e^{-i\varphi_m})^k \exp(-ik\omega_1 t) \quad (m = 1, 2, \ldots, N). \tag{4.2}$$

Due to the linearity of the problem, we shall represent the nonstationary components of circulations at the profiles of the grid as a sum of the same harmonics analogously to (3.10):

$$\Gamma_n = -\frac{2}{N} \sum_{k=1}^{N-1} \delta_k \bar{\varepsilon}^k \{q \sin[k(\omega_1 t + \theta_n) + \beta_k] + \Gamma_0 \cos[k(\omega_1 t + \theta_n) + \beta_k]\}, \tag{4.3}$$

where δ_k, β_k are the amplitude coefficient and shift in phase of the k-th harmonic, arising due to the effect of the vortex wakes.

To determine the values of δ_k and β_k, it is necessary to find the position of the lines of the contact discontinuity $\mathscr{L}_n$, simulating the vortex wakes, and the intensity of the vortices in these wakes at any moment in time. For this purpose, we shall give the position of some free elementary vortex, converging with the n-th profile, in the form

$$\zeta_n(\tau) = \rho(\tau) e^{i\psi_n(\tau)},$$

where τ is the parameter determining the time interval from the moment that this vortex separates from the profile. Assuming that the vortices connected to the n-th profile are concentrated at the point $z_n = r \exp[i(\theta_n + \omega t)]$, the angular coordinate of the free vortex can be determined from the equation

$$\psi_n(\tau) = \theta_n + \omega t - \omega\tau + \bar{\psi}, \tag{4.4}$$

where $\bar{\psi}$ is the angle of inclination of the vortex due to its absolute motion in the circular direction.

Using the expressions for the projections of the absolute velocity of the motion of a vortex on the radial and circular direction

$$v_\rho = d\rho/d\tau = q/(2\pi\rho),\ v_\psi = \rho\partial\tilde{\psi}/\partial\tau = (\Gamma_0 + N\gamma_0)/(2\pi\rho),$$

and taking into account the initial conditions $\rho = r$, $\tilde{\psi} = 0$ at $\tau = 0$, we find

$$q\tau/\pi = r^2(\bar{\rho}^2 - 1),\ \tilde{\psi} = \bar{\Gamma}_0 \ln \bar{\rho}, \tag{4.5}$$

where

$$\bar{\rho} = \rho/r;\ \bar{\Gamma}_0 = (\Gamma_0 + N\gamma_0)/q.$$

With the help of (4.5), expression (4.4) transforms into the form

$$\psi_n = \theta_n + \omega t - \bar{\psi}(\bar{\rho}), \tag{4.6}$$

where $\bar{\psi} = (c/2)(\bar{\rho}^2 - 1) - \bar{\Gamma}_0 \ln \bar{\rho}$; $c = (2\pi r^2\omega)/q$.

The dependence (4.6) represents the equation of the line of the vortex wake $\mathscr{L}_n$, converging on the n-th profile, for the given time t.

The vortex intensity per unit length in the wake, owing to the change in circulation around the n-th profile with respect to the k-th harmonic, can be determined from equation [3]

$$\gamma_n^{(k)}(s, t) = -\frac{1}{v(s)} \frac{\partial\Gamma_n^{(k)}}{\partial t}\bigg|_{t=t_1} \quad (t_1 = t - \tau), \tag{4.7}$$

where v(s) is the relative velocity of vortices in a system of coordinates fixed rigidly to the grid; s is the arc length along the vortex line.

We shall now examine the expression for the intensity of an elementary free vortex with an arc-length coordinate s:

$$d\bar{\Gamma}_n^{(k)}(s) = \gamma_n^{(k)}(s)\, ds.$$

Since $ds = v(s)d\tau$, taking into account the fact that s and τ are functions of the parameter ρ, we obtain with the help of (4.5) and (4.7)

$$d\bar{\Gamma}_n^{(k)}(\rho) = -\frac{2\pi\rho}{q} \frac{\partial\Gamma_n^{(k)}}{\partial t}\bigg|_{t=t_1} d\rho = \tilde{\gamma}_n^{(k)}(\rho)\, d\rho.$$

We find the expression for $\tilde{\gamma}_n^{(k)}$ with the help of (4.3)

$$\tilde{\gamma}_n^{(k)} = \frac{4\pi k\omega_1\bar{\varepsilon}^k\delta_k}{qN} \rho\, \{q \cos[k(\omega_1 t + \theta_n) + \beta_k] - \Gamma_0 \sin[k(\omega_1 t + \theta_n) + \beta_k]\}.$$

Thus, as a result of the transformations performed, the position of the vortex wakes and their intensity per unit length are determined as functions of the parameters ρ and t. This permits determining quite simply the nonstationary component of the complex fluid flow velocity taking into account the vortex wakes. For the k-th harmonics of the connected and free vortices of all profiles in the grid, it will equal

$$\bar{v}^{(k)}(z) = \frac{1}{2\pi i} \sum_{n=1}^{N} \left[\frac{\Gamma_n^{(k)}}{z - z_n} + \int_r^\infty \frac{\tilde{\gamma}_n^{(k)} d\rho}{z - \zeta(\rho)} \right].$$

It can be shown that for $N \to \infty$ the complex velocity $\bar{v}^{(k)}(z)$ will approach its limiting value, which equals the velocity induced by the connected and free vortices, continuously distributed along the circle, i.e.,

$$\lim_{N\to\infty} \bar{v}^{(k)}(z) = \tilde{v}^{(k)}(z) = \frac{N}{(2\pi)^2 i} \oint \left[\frac{\bar{\gamma}^{(k)}(\theta)}{z - re^{i(\theta+\omega t)}} + \int_r^\infty \frac{\tilde{\gamma}(\theta, \rho)\, d\rho}{z - \rho e^{i(\theta+\omega t - \bar{\psi})}} \right] d\theta. \tag{4.8}$$

Here, $\bar{\gamma}^{(k)}(\theta)$ and $\tilde{\gamma}^{(k)}(\rho, \theta)$ are continuous functions, coinciding at $\theta = \theta_n$ with the values $\frac{N}{2\pi r}\Gamma_n^{(k)}(\theta_n)$ and $\frac{N}{2\pi r}\tilde{\gamma}^{(k)}(\theta_n)$.

Calculating the contour integrals in expressions (4.8) using the theory of residues, we find the values of $\tilde{v}^{(k)}$ at the control points of the profiles:

$$\widetilde{v}^{(k)}(z_m) = -\frac{\bar{\varepsilon}^k\delta_k \exp[-i(\omega t+\varphi_m)]}{2\pi i r}\left\{\frac{q-i\Gamma_0}{\bar{R}^{k+1}}\exp[-ik(\omega_1 t+\varphi_m)-i\beta_k]\left(i-kc\bar{\omega}_1 A_1^{(k)}\right)\right. \tag{4.9}$$
$$\left.+kc\bar{\omega}_1(q+i\Gamma_0)\bar{R}^{k-1}\exp[ik(\omega_1 t+\varphi_m)+i\beta_k]A_2^{(k)}\right\},$$

where

$$A_1^{(k)} = \int_1^{\bar{R}} \bar{\rho}^{k+1}\exp\left\{-ik\left[\frac{c}{2}\bar{\omega}_0(\bar{\rho}^2-1)-\bar{\Gamma}_0\ln\bar{\rho}\right]\right\}d\bar{\rho},$$
$$A_2^{(k)} = \int_{\bar{R}}^{\infty}\frac{1}{\bar{\rho}^{k+1}}\exp\left\{ik\left[\frac{c}{2}\bar{\omega}_0(\bar{\rho}^2-1)-\bar{\Gamma}_0\ln\bar{\rho}\right]\right\}d\bar{\rho} \tag{4.10}$$
$$\left(\bar{\omega}_0=\frac{\omega_0}{\omega},\quad \bar{\omega}_1=\frac{\omega_1}{\omega},\quad \bar{R}=\frac{R}{r}\right).$$

We introduce the notation

$$q-i\Gamma_0=\bar{q}e^{-i\xi},\quad A_1^{(k)}=g_1^{(k)}e^{i\eta_1^{(k)}},\quad A_2^{(k)}=g_2^{(k)}e^{i\eta_2^{(k)}}. \tag{4.11}$$

From the impenetrability condition (3.2), we obtain using (4.2) and (4.9)

$$\operatorname{tg}\beta_k = c\bar{\omega}_1\left\{\frac{g_1\cos\eta_1+g_2\bar{R}^{2k}\cos(2\alpha_1+\eta_2)}{1-c\bar{\omega}_1 g_1\sin\eta_1+c\bar{\omega}_1 g_2\bar{R}^{2k}\sin(2\alpha_1+\eta_2)}\right\}^{(k)}, \tag{4.12}$$
$$\delta_k=\left\{\frac{2}{\cos\beta_k+c\bar{\omega}_1[g_1\sin(\beta_k+\eta_1)+g_2\bar{R}^{2k}\sin(2\alpha_1+\eta_2+\beta_k)]}\right\}^{(k)}.$$

Analogously to (4.9) we find the value of $\widetilde{v}^{(k)}$ at the point ε:

$$\widetilde{v}^{(k)}(\varepsilon) = -\frac{r}{2}c\sqrt{1+\operatorname{tg}^2\xi}\,\delta_k b_k\bar{\varepsilon}^{2k-1}\exp[i(\beta_k+\gamma_k+\xi-\omega_0 t)], \tag{4.13}$$

where $b_k e^{i\gamma_k}=1-ic\bar{\omega}_1 A_3^{(k)}$; $A_3^{(k)}=\int_1^{\infty}\frac{1}{\bar{\rho}^{k-1}}\exp\left\{ik\left[\frac{c}{2}(\bar{\rho}^2-1)-\bar{\Gamma}_0\ln\bar{\rho}\right]\right\}d\bar{\rho}.$

Expression (4.13) together with (4.10)–(4.12) determines the complex velocity induced by connected and free vortices, at the location of the vortex source as a function of coordinates of this location and time.

Summing all harmonics of this velocity and satisfying Eq. (2.1), using (3.3) and (4.1) we find the relations that must be satisfied by the parameters of the limit cycle $\bar{\varepsilon}$, $\bar{\omega}_0$ taking into account the influence of the vortex wakes:

$$\sum_{k=1}^{N-1}\delta_k b_k\bar{\varepsilon}^{2(k-1)}\cos(\beta_k+\gamma_k+\xi)=0,$$
$$\bar{\omega}_0=\frac{c}{2}\sqrt{1+\operatorname{tg}^2\xi}\sum_{k=1}^{N-1}\delta_k b_k\bar{\varepsilon}^{2(k-1)}\sin(\beta_k+\gamma_k+\xi).$$

5. Limit Cycle. We shall show that within the framework of the model examined in Sec. 4, the limit cycle of the motion of the vortex source exists, at least, for small values of its parameters

$$\bar{\omega}_0 \ll 1,\ \bar{\varepsilon}\ll 1. \tag{5.1}$$

Assuming that $\delta_k \sim 1$ and $b_k \sim 1$ for $k > 1$, in the first equation we need only consider the two terms in the sum, and in the second only three terms. Then, we shall have

$$\bar{\varepsilon}^2 \approx -\frac{\delta_1 b_1\cos\chi_1}{\delta_2 b_2\cos\chi_2}\quad(\chi_k=\beta_k+\gamma_k+\xi); \tag{5.2}$$

$$\bar{\omega}_0\approx\frac{c}{2}\sqrt{1+\operatorname{tg}^2\xi}\left[\delta_1 b_1\frac{\sin(\chi_1-\chi_2)}{\cos\chi_2}+\bar{\varepsilon}^4\delta_3 b_3\sin\chi_3\right]. \tag{5.3}$$

Here δ_k, β_k, b_k, and γ_k are functions of the quantity $\bar{\omega}_0$ sought and the starting parameters c, Γ_0, α_1, and ξ, $\bar{R}$. They are determined with the help of the integrals $A_j^{(k)}$ (4.10) and (4.13). We shall examine these integrals.

Let $\bar{R}-1\ll 1$. Then for $A_1^{(k)}$ we have the estimate

$$A_1^{(k)}\ll 1\quad(k=1,2,3). \tag{5.4}$$

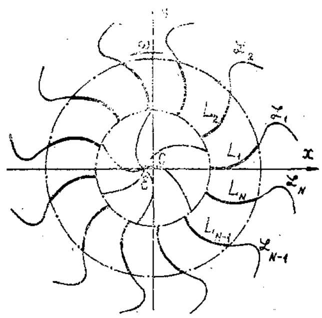

Fig. 3

The improper integrals $A_j^{(k)}$ (j = 2, 3) are calculated with the help of tables [4] and have the form

$$A_j^{(k)} = \frac{1}{2}\exp\left[i\left(\frac{\pi}{2}\nu_k - \lambda_j^{(k)}\right) - \nu_k \ln\left(\lambda_j^{(k)}\right)\right]\Gamma\left(\nu_k, -i\bar{\rho}_j\lambda_j^{(k)}\right) \quad (k = 1, 2, 3), \tag{5.5}$$

where $\Gamma(\nu, \lambda)$ is a gamma function;

$$\nu_k = 1 - k(1 + i\bar{\Gamma}_0)/2;\ \bar{\rho}_2 = \bar{R}^2;\ \bar{\rho}_3 = 1;$$
$$\lambda_2^{(k)} = kc\bar{\omega}_0/2;\quad \lambda_3^{(k)} = kc/2.$$

Here, the expressions for $A_2^{(k)}$ can be represented approximately as function of the parameter $\bar{\omega}_0$:

$$A_2^{(1)} \approx \frac{1}{\sqrt{\bar{\omega}_0}} f_1 \exp\left(i\frac{\bar{\Gamma}_0}{2}\ln\bar{\omega}_0\right),\ A_2^{(2)} \approx f_2 \exp\left(i\bar{\Gamma}_0 \ln\bar{\omega}_0\right),\ A_2^{(3)} \approx f_3, \tag{5.6}$$

where f_k are complex functions of the parameters c and $\bar{\Gamma}_0$. The values of $A_3^{(k)}$, as also the values of f_k, do not depend on $\bar{\omega}_0$.

Using (5.4), we obtain from (4.12)

$$\beta_k = \frac{\pi}{2} - 2\alpha_1 - \eta_2^{(k)} - \varkappa_k,\quad \delta_k = \frac{2}{\cos\beta_k + ca_2^{(k)}\cos\varkappa_k}, \tag{5.7}$$

where $\varkappa_{(k)} = \arcsin\left(\frac{\sin\beta_k}{ca_2^{(k)}}\right)$. In the case c ~ 1, it follows from (5.5)-(5.7) that

$$\varkappa_1 \ll 1,\quad \delta_1 = 2\sqrt{\bar{\omega}_0}/c\,|f_1|. \tag{5.8}$$

If now (5.8) is substituted into (5.2) and (5.3), then we obtain

$$\bar{\varepsilon}^2 = -h_1\sqrt{\bar{\omega}_0}\frac{\cos\chi_1}{\cos\chi_2}; \tag{5.9}$$

$$\sqrt{\bar{\omega}_0}\left[1 - h_2\left(\frac{\cos\chi_1}{\cos\chi_2}\right)^2\right] = h_3\frac{\sin(\chi_1 - \chi_2)}{\cos\chi_2}, \tag{5.10}$$

where h_n are some functions of the parameters c and $\bar{\Gamma}_0$. Further, keeping in mind the fact that γ_k does not depend on $\bar{\omega}_0$, we find with the help of (5.2) and (5.6)-(5.8)

$$\chi_1 - \chi_2 = \chi_0 + \varkappa_2 - (\bar{\Gamma}_0/2)\ln\bar{\omega}_0, \tag{5.11}$$

where χ_0 is some bounded function of the parameter c and $\bar{\Gamma}_0$.

It follows from (5.11) that the solution of the system (5.9) and (5.10) exists in a wide range of values of starting parameters c, $\bar{\Gamma}_0$, α_1, and ξ, with the exception, possibly of some discrete set of values. However, the range of values of these parameters is limited by the condition of stability of the limit cycle, which follows from (4.13) taking into account (5.1) and has the form $\cos\chi_1 < 0$.

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